**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

The total time available for servicing the car is 60 - 10 = 50 minutes. mean μ = 45 minutes and standard deviation σ = 8 minutes.

Z = (X - μ) / σ, X is the time required for servicing the transmission.

Z = (50 - 45) / 8 = 0.625

1-stats.norm.cdf(abs(0.625))

=0.265

The probability that the service manager cannot meet his commitment is 26.76%(option B).

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer:

p(X>44)- Employees older than 44 yrs of age

1-stats.norm.cdf(44,loc=38,scale=6)

0.15865525393145707

p(38<X<44); Employees between 38 to 44 yrs of age

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

0.3413447460685429

Therefore, we can say that the employees older than 44 are fewer than the employees between 38 and 44. Hence, it is False.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

P(X<30); Employees under 30 yrs of age

stats.norm.cdf(30,38,6)

0.09121121972586788

No. of employees attending training program from 400 nos. is N\*P(X<30)

400\*stats.norm.cdf(30,38,6)

36.484487890347154

The statement is True. A training program for employees under the age of 30 at the center would be expected to attract 36 employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer: The difference between 2X1 and X1 + X2 is= 2X1 - (X1 + X2) = X1 - X2

Since X1 and X2 are independent and normally distributed, their difference X1 - X2 is also normally distributed with below:

mean:

E(X1 - X2) = E(X1) - E(X2) = μ - μ = 0

variance:

Var(X1 - X2) = Var(X1) + Var(X2) = σ^2 + σ^2 = 2σ^2

Therefore, X1 - X2 follows a normal distribution with mean 0 and variance 2σ^2.

On the other hand, 2X1 and X1 + X2 are also normally distributed with below:

Means and variances:

E(2X1) = 2μ, Var(2X1) = 4σ^2

E(X1 + X2) = E(X1) + E(X2) = 2μ, Var(X1 + X2) = Var(X1) + Var(X2) = 2σ^2

So, 2X1 follows a normal distribution with mean 2μ and variance 4σ^2, while X1 + X2 follows a normal distribution with mean 2μ and variance 2σ^2.

In summary, the difference X1 - X2 follows a normal distribution with mean 0 and variance 2σ^2, while 2X1 and X1 + X2 both follow normal distributions with mean 2μ, but with variances of 4σ^2 and 2σ^2, respectively.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer: The normal distribution is symmetric about its mean= 100.

stats.norm.interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer: Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer: 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

For the first division=0.0477903522728147

For the second division=0.040059156863817086

First Division has the larger probability